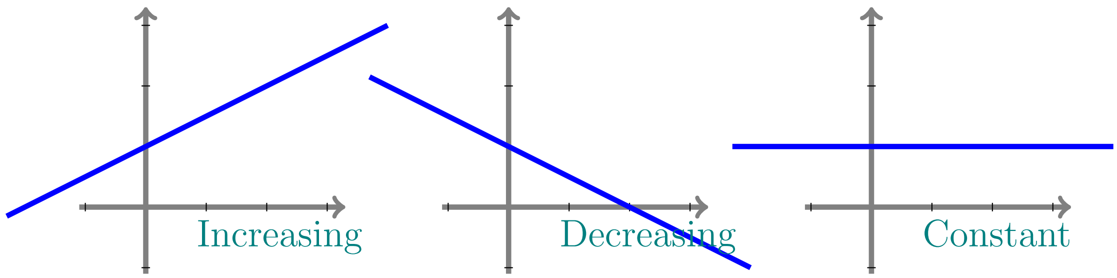
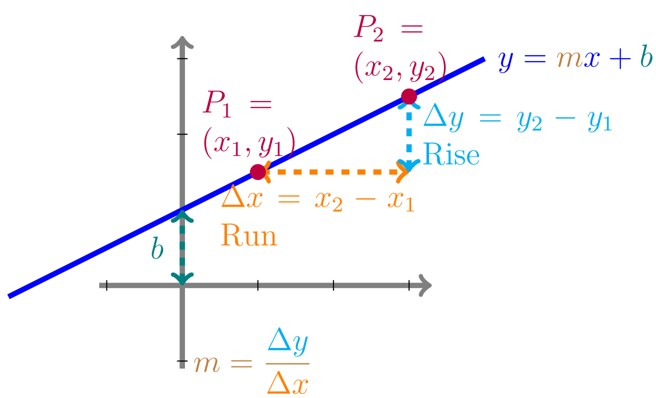
2.1-2.2: Linear Functions and Their Graphs

Linear Functions

- A linear function is a function whose **average rate of change** is **constant**. The general form of a linear function is $f(x) = mx + b$ where m is the constant rate of change. If $m > 0$, then the linear function is increasing. If $m < 0$, then the linear function is decreasing. If $m = 0$, then the function is a constant function.
- The graph of a linear function is a line.



Rate of Change, Rise, Run and Slope



- $y = \underbrace{m}_{\text{Slope}} x + \underbrace{b}_{y\text{-intercept}}$ is called the **slope intercept form** of a line.
- Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{Rise}}{\text{Run}}$ and b is the y-value of point with x-value$= 0$.
- Note that $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$ so the order of choosing points P_1 and P_2 doesn't matter in calculating the slope as long as the same order is preserved for numerator and the denominator.

Finding the Equation of a Line

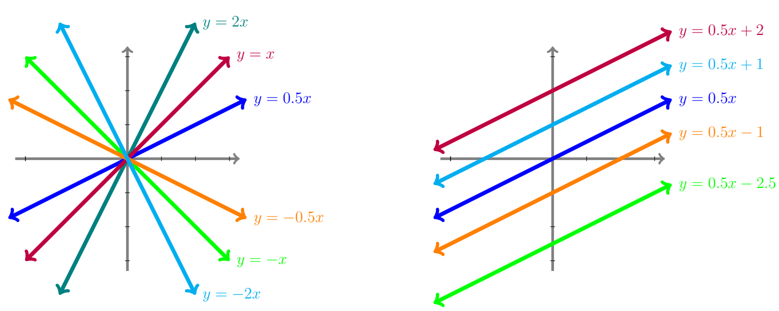
- Find the slope m. If two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are given, then use the formula
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$
- Use one of the points and the slope you found to write the **point-slope** equation:
$$y - y_1 = m\,(x - x_1)\,.$$
- Use the point-slope form to derive the slope-intercept form:
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - y_1 &= mx - mx_1 \\ y &= mx - mx_1 + y_1 \\ y &= mx + \underbrace{(y_1 - mx_1)}_{b} \end{aligned}$$

Business

Let x be the number of units of product being produced and sold.

- The revenue function, $R(x)$, represents the total sale.
- The cost function, $C(x)$, represents the total cost.
- Profit function is the difference function $P(x) = R(x) - C(x)$.
- Because of the fixed cost $C(x) > R(x)$ for small x and it is expected that after certain number of the sales, the process reverses, $R(x) > C(x)$. The point when the process reverses is the breaking even point.

Comparing Slopes

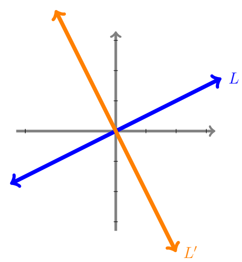


Different slopes but same y-intercept. Same slopes but different y-intercept.

- Vertical lines are of the form $x = c$ where c is a constant. Vertical lines are not a graph of a function of x.
- Horizontal lines are of the form $y = c$ where c is a constant.
- Consider two linear functions $f(x) = mx + b$ and $g(x) = m'x + b'$. If $m \neq m'$, then the two lines intersect at a point, denote it by $A(p, q)$. Additionally, if $m > m'$, then for all $x < p$, $f(x) < g(x)$ and for all $x > p$, $f(x) > g(x)$.
- Consider two linear functions $f(x) = mx + b$ and $g(x) = m'x + b'$. If $m = m'$, then the two lines are parallel.

Perpendicular Lines

- If the lines L and L' are perpendicular ($L \perp L'$) and m is slope of L and m' is the slope of L', then $m \cdot m' = -1$. That is, $m' = \frac{-1}{m}$.



1. Let L be line through points $(2, 3)$ and $(5, 1)$.

   (a) What is the slope of line L?

   (b) Is the line, L, the graph of an increasing or a decreasing function?

   (c) What is the equation of line L ?

   (d) What is the y-value for a point on the line if the x-value is 6?

2. **Business and Econ:** A manufacturer is estimating that the cost in material, labor and utility for producing one hat is \$5. The fixed cost of keeping the hat factory open, such as rent and different subscriptions to utilities, is $\$20,000$ a month. If monthly production is x units of hats, express the total monthly cost of the manufacturer as a function of units produced.

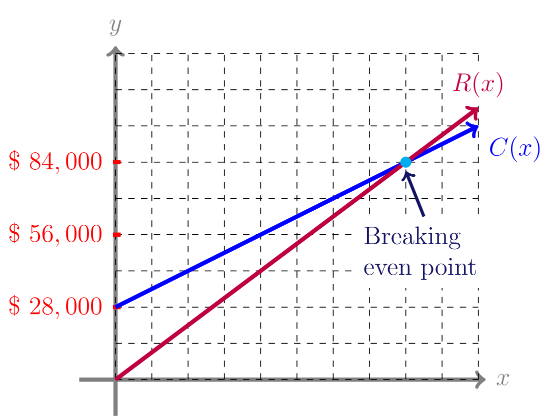
3. **Business and Econon:** A manufacturer is estimating that the total cost of producing x units of their product is $C(x) = 28,000 + 0.5x$ dollars and each unit sells for 75 cents.

   (a) Express the revenue in dollars of selling x units of the product as a function of x. (Note that the revenue is the total income.)

   (b) At what value for x, does the revenue equal to the total cost?

   (c) Express the profit in dollars of selling x units of the product as a function of x.

   (d) For what values of x, does the manufacturer earn profit?



4. (a) Find an equation for the line L passing through the points $(-3, 7)$ and $(1, -7)$.

(b) Find an equation for the line **perpendicular** to L and passing through point $(0, 5)$.

5. (a) Find an equation for the line L passing through the points $(-3, 7)$ and $(1, 7)$.

(b) Find an equation for the line **perpendicular** to L and passing through $(10, 5)$.

6. Which of the following lines are **parallel** to line $y = -\frac{2}{7}x + 3$.

(a) $y + \frac{2}{7}x = 1$

(b) $y - \frac{2}{7}x = 1$

(c) $7y = 2x + 21$

(d) $7y = -2x + 3$

(e) $7y + 2x = 7$

(f) $7y - 2x = 2$

(g) $2y - 7x = 3$

(h) $2y + 7x = -21$

7. Which of the following lines are **perpendicular** to line $y = -\frac{5}{7}x + 3$.

(a) $y + \frac{5}{7}x = 3$

(b) $y - \frac{7}{5}x = 3$

(c) $5y = 7x + 21$

(d) $7y = -5x + 7$

(e) $7y + 5x = 5$

(f) $7y - 5x = 3$

(g) $5y + 7x = 21$

(h) $5y - 7x = -5$

8. The line segment in the figure shown to the right is a portion of the line whose equation is

(a) $y = \frac{3}{5}x + 1$

(b) $y = \frac{3}{5}x + 4$

(c) $y = \frac{-1}{5}x + 1$

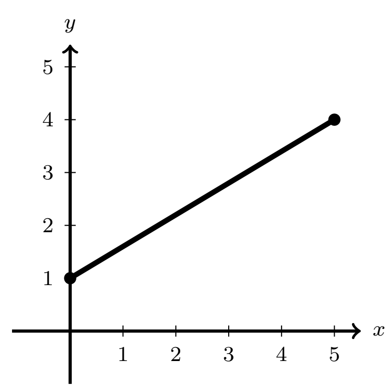
(d) $y = \frac{-1}{5}x + 4$

(e) $y = \frac{-3}{5}x + 1$

(f) $y = \frac{-3}{5}x + 4$

(g) $y = \frac{1}{5}x + 1$

(h) $y = \frac{1}{5}x + 4$



9. Graph

$$f(x) = \begin{cases} x & x < -2 \\ 0.5x & -2 \le x \le 0 \\ -2x & 0 < x < 4 \\ 3x - 16 & x \ge 4 \end{cases}.$$

Label two points of each linear piece of graph.